Correlated Waveform Design:
A Step Towards a Software Radar

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• Radar Group @ KAUST
• What is a Radar?
• Beampatterns of Fixed Antennas
• Beampatterns Using Antenna Array
• The Concept of MIMO Radar
• Waveform Covariance Matrix Design
• Correlated Waveform Design
• Group Publications
Radar Group @ KAUST

- **Group Leader:** Prof. Mohamed-Slim Alouini (FIEEE)
- **Co-Investigator:** Prof. Tareq Naffouri
- **Co-Investigator:** Dr Sajid Ahmed (Research Scientist)
- **PhD Students**
  - Seifallah Jardak
  - Hussain Ali (KFUPM + KAUST)
- **MSc Students**
  - Taha Bachoucha
  - *John Lipor* (PhD Student at University of Michigan Ann Arbor)
- **Intern Students**
  - *Abdulrahman Alsaggaf* (KFUPM)
  - *Ayman Magrabi* (KFUPM)
• **Major Research Directions:**
  - Covariance Matrix Design for Linear and Planar Beampatterns
  - Finite Alphabet Correlated Waveform Design
  - Target Parameter Estimation
  - Detection Using Compressive Sensing Algorithms
  - Development of Prototype Radar

• **Strengths:**
  - Radar System Modelling
  - Novel Closed-Form Solutions for Covariance Matrices Design
  - Novel Closed-Form Solutions for Waveform Design
  - Low Complexity Target Parameter Estimation Techniques
Radar Group @ KAUST

• Accomplishments (since Feb. 2012):
  – Four Journal and six conference papers
  – Three Journal and four conference papers are under review
  – One patent (invention disclosure) filed
  – One grant from OCRF of KAUST
  – One proposal under review in KACST, Riyadh, Saudi Arabia
  – One proposal under review in QNRF, Doha, Qatar
What is a Radar? (1/2)

- Radar stand for RAdio Detection And Ranging.

- Radar is an object detection system that uses electromagnetic waves to detect the range, location and radial speed of both moving and stationary targets such as
  - Aircraft
  - Ship
  - Motor vehicle
  - Cloud

- Generally radar has two basic parts, a transmitter and a receiver, which are usually collocated.
What is a Radar? (2/2)

Figure 1: Basic radar system.
An Isotropic antenna transmits power in all directions equally.

To increase the range and reflected power, the power is transmitted only in the region-of-interest.
Beampattern of Parabolic Antenna

- Parabolic antenna focus the transmitted power in one direction.
- To scan the target at different locations the antenna platform is mechanically rotated.

Figure 2: Parabolic antenna.

Figure 3: Beampattern of parabolic antenna.
In an antenna array $M$ antennas are used to transmit the signals.

The received signal at an angle $\psi$ can be written as

$$r(n, \psi) = \sum_{m=1}^{M} x_m(n) e^{j\pi(m-1)\sin(\psi)} = a_T^T(\psi)x(n),$$

where

$$a_T(\psi) = [1\ e^{j\pi\sin(\psi)}\ \ldots\ e^{j\pi(M-1)\sin(\psi)}]^T,\ x(n) = [x_1(n)\ x_2(n)\ \ldots\ x_M(n)]^T.$$
The power received at an angle $\psi$ can be found as

$$
P(\psi) = E\{|a_T^T(\psi)x(n)|^2\},$$

$$= E\{a_H^H(\psi)x(n)x(n)^H a_T(\psi)\},$$

$$= a_T^H(\psi) E \{x(n)x(n)^H\} a_T(\psi),$$

$$= a_T^H(\psi) Ra_T(\psi),$$

where $R$ is the correlation or covariance matrix of the waveform.
Phased array radar focus the transmitted power in the given direction.

Figure 4: Phased-array radar.

\[
r(n, \psi) = \sum_{m=1}^{M} x(n) e^{-j\pi(m-1) \sin(\psi)} e^{j\pi(m-1) \sin(\psi)}.
\]

\[
P(\psi) = a_T^H(\psi) R a_T(\psi).
\]
• The transmitted waveform from different antennas in vector form can be written as

\[
x(n) = \left[ x(n) \ x(n)e^{-j\pi \sin(\psi)} \ \cdots \ x(n)e^{-j\pi(M-1) \sin(\psi)} \right]^T.
\]

• In phased-array radar the covariance matrix of the transmitted waveforms is

\[
R = E\{x(n)x^H(n)\},
\]

\[
= \begin{bmatrix}
1 & e^{j\pi \sin(\psi)} & \cdots & e^{j\pi(M-1) \sin(\psi)} \\
e^{-j\pi \sin(\psi)} & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & e^{j\pi \sin(\psi)} \\
e^{-j\pi(M-1) \sin(\psi)} & \cdots & e^{-j\pi \sin(\psi)} & 1
\end{bmatrix}.
\]
- Phased array radar focus the transmitted power in the given direction

Figure 5: Beampattern of Phased Array radar for $\psi = 20$ degrees.
• In MIMO-radars the waveforms can be independent or partially correlated

Figure 6: MIMO-radar.
• In MIMO-radar the waveforms can be independent

\[ R = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & 1
\end{bmatrix} \]

• Waveforms can be partially correlated

\[ R = \begin{bmatrix}
1 & \rho_{12} & \cdots & \rho_{1M} \\
\rho_{21} & 1 & \ddots & \vdots \\
\vdots & \ddots & \ddots & \rho_{M-1,M} \\
\rho_{M1} & \cdots & \rho_{M,M-1} & 1
\end{bmatrix} \]

• MIMO-radar yields \( \frac{M^2 - M}{2} \) degree-of-freedom
The transmitted power from MIMO-radar at the location $\theta_k$ is given by

$$P(\theta_k) = a^H(\theta_k)Ra(\theta_k).$$

To synthesize $R$ for the desired beampattern, $\phi(\theta_k)$, the cost functions can be defined as

$$J_1(R) = \frac{1}{K} \sum_{k=1}^{K} \left| a^H(\theta_k)Ra(\theta_k) - \alpha\phi(\theta_k) \right|^2,$$

subject to the constraints

$$C_1. \ R \geq 0$$

$$C_2. \ R(m, m) = c, \ \text{for} \ m = 1, \ldots, M$$
Figure 7: Symmetric beampattern of 60 degrees. The number of transmit antenna is 10.
Figure 8: Beampattern of two main lobes. The number of transmit antennas is 20.
Figure 9: Non symmetric beampattern of 40-degrees width. The number of transmit antenna is 10.
Correlated Waveforms Matrix Design

To reduce the side-lobe-levels

\[ J_1(R) = \frac{1}{K} \sum_{k=1}^{K} \left| a^H(\theta_k)Ra(\theta_k) - \alpha \phi(\theta_k) \right|^2, \]

- To control main and side-lobe-levels following constraints can be added in the problem

  \begin{align*}
  C_1 & . \quad R \geq 0 \\
  C_2 & . \quad R(m, m) = c, \text{ for } m = 1, \ldots, M \\
  C_3 & . \quad \max\{a^H(\theta_m)Ra^H(\theta_m)\} - \min\{a^H(\theta_m)Ra^H(\theta_m)\} \leq \delta \\
  C_4 & . \quad \max\{a^H(\theta_s)Ra^H(\theta_s)\} \leq \epsilon.
  \end{align*}

- Convex optimisation toolbox of Matlab can be used to optimise the covariance matrix R.
Figure 10: Demonstration of direct ripple control using constraint $C_3$ for various values of $\delta$, where the region of interest is $\theta_p \in [-30^\circ, 30^\circ]$ and the number of transmit antennas is 10.
Figure 11: Demonstration of direct ripple control using constraint $C_4$ for various values of $\epsilon$, where the region of interest is $\theta_s \in [-90^\circ, -60^\circ] \& [60^\circ, 90^\circ]$ and the number of transmit antennas is 40.
**Finite Alphabet Correlated Waveform Design (1/2)**

- Once the covariance matrix $\mathbf{R}$ is found, $M$ waveforms each of $L$ symbols, $\mathbf{X} = [x_1 \ x_2 \ \cdots \ x_M] \in \mathbb{C}^{L \times M}$ to realise it can be found as

$$\mathbf{X} = \mathcal{X}_g \mathbf{R}^{1/2}. \quad (1)$$

![Figure 12: Waveforms using (1) and desired amplitude waveform.](image)
Finite Alphabet Correlated Waveform Design (2/2)

- Gaussian random variable, \( x \), can be mapped onto BPSK random variable, \( y \), using the relation
  \[
  y = \text{sign}(x).
  \]

- The relationship between the covariance matrix of Gaussian and BPSK RV’s can be established as
  \[
  R_g = \sin \left( \frac{\pi}{2} R \right).
  \]

- The matrix of Gaussian RV’s can be found with de-whitening transform
  \[
  X = \mathcal{X}_g R_g^{1/2}.
  \]

- The matrix of desired BPSK waveform can be obtained as
  \[
  Y = \text{sign} \left( X \right).
  \]


